# How modules hint at $\infty$-categories

**Def.**: The bicategory $\mathbb{M}\mathrm{od}$ has:

- as objects, rings
- as 1-morphisms, bimodules
- as 2-morphisms, bimodule isomorphisms.

Composition of 1-morphisms is given by the tensor product of bimodules.
Identity 1-morphism of an object $R$ is given by $R$ as an $R$-$R$-bimodule.

**Remark**: $R \simeq S$ in $\mathbb{M}\mathrm{od}$ iff $R$ and $S$ are Morita equivalent. We like Morita equivalence.

**Problem?**: A priori, the tensor product of two bimodules does not exist, only up to unique isomorphism.

**Solution?**: We choose a tensor product for all pairs of composable bimodules.

But we don't like choices.

**Q.**: Is there a version of category where composition doesn't need to exist uniquely?
**A.**: Yes, $\infty$-categories!

In an $\infty$-category, the composite of two morphisms only exists up to contractible choice. Any such choice witnesses the composition.
Topological analogue: The concatenation of two paths only exists up to homotopy.

**Conclusion**: It is useful to think of $\mathbb{M}\mathrm{od}$ as an $\infty$-category.